on a particle; g, acceleration due to gravity; k, wave vector; k_0, maximum value of k determined in (5); K, function describing the effect of physical constraint on the viscous force of the phase interaction; m, mass of particle; n, numerical concentration of particles; p, pressure; u, mean relative velocity of the phases; v, w, velocities of the fluid and particles; V_i, W_i, dimensionless fluctuation velocities; $\delta_0 = E_2/E_1$; $\kappa = d_1/d_0$; μ, ν, absolute and kinematic viscosities; ρ, ρ_*, volumetric concentration of particles and the concentration corresponding to the packing density; ω, frequency; Φ, Ψ, spectral densities. The subscripts 0 and 1 respectively denote the fluid and particles; the primes denote fluctuation quantities; the brackets denote averaging.

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NUMERICAL MODELING OF TURBULENT FIELDS OF VELOCITY, TEMPERATURE, AND CONCENTRATION IN A RECTANGULAR CHANNEL

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The finite elements method is used to solve a system of three-dimensional transport equations in a rectangular channel.

Introduction. The numerical modeling of flow and heat and mass transfer in complicated channels is a new and important means of investigation. The popularity of numerical experiments employing computers stems from many factors, the most important being the completeness of the information, the speed with which it is obtained, and the possibility of modeling a wide range of situations – including some that cannot be realized in a physical experiment.

The mathematical model and application package employed in the present article can be used with success to describe and model a wide range of so-called parabolic flows in closed channels of complex form. At the current stage of investigation, we will restrict ourselves to flows without buoyancy and we will consider only the longitudinal component of the velocity vector. The model contains two assumptions connected with Newtonian fluids. The system of transport equations is based on the equations of continuity, motion, energy, diffusion, and heat conduction. The numerical realization was accomplished on the basis of the finite elements method in the Galerkin modification. Below, we report the details of the formulation and method of solution of the problem. We also generalize the results and present certain other results of numerical experiments.

Mathematical Model. Initial Equations. To describe the hydrodynamic part of the problem, we will use the model of parabolic flow in a closed rectangular channel (Fig. 1). the Navier–Stokes equations for the longitudinal component of the velocity vector have the form:

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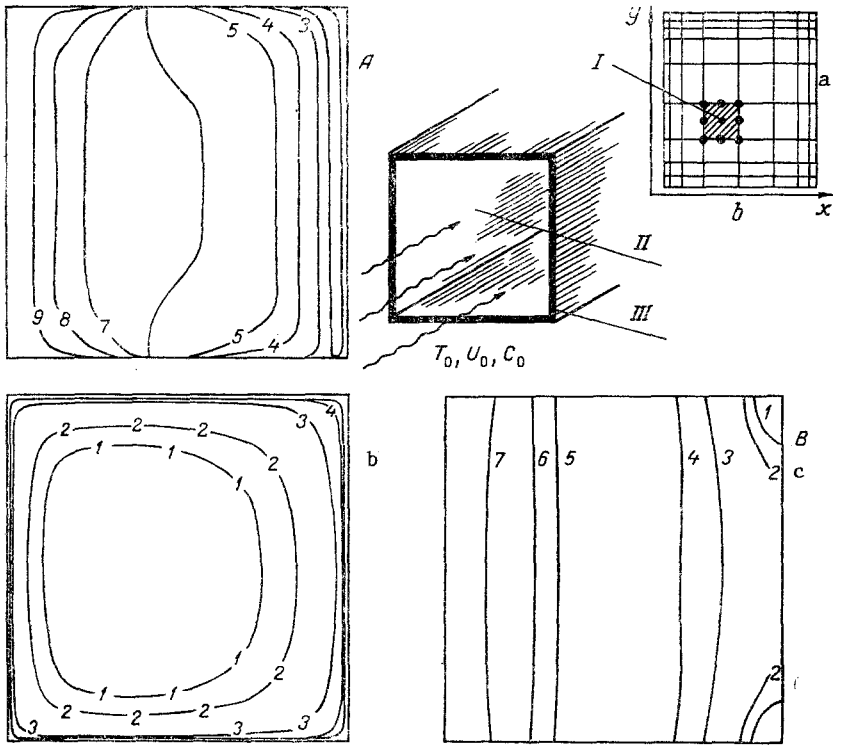

Fig. 1. Diagram of the flow, the region of integration, and isolines of velocity, temperature, and concentration. A) Isolines of T: 1) 350, 2) 380, 3) 430, 4) 450, 5) 480, 6) 490, 7) 500, 8) 520, 9) 610. B) Isolines of U: 1) 1.0, 2) 0.85, 3) 0.60, 4) 0.30, 5) 0.20. C) Isolines of C: 1) 0.002, 2) 0.0021, 3) 0.0025, 4) 0.003, 5) 0.008, 6) 0.01; 7) 0.015). I) Isoperimetric square; form of the equation being solved: II) $A(\partial\Phi/\partial z) = \nabla(\Gamma\nabla\Phi) + Q$; III) $\nabla(\lambda\nabla T) = 0$; T_0, U_0, C_0) input values.

$$\rho U\frac{\partial U}{\partial z} = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}\left[(\mu+\mu_t)\frac{\partial U}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\mu+\mu_t)\frac{\partial U}{\partial y}\right]. \tag{1}$$

The continuity equation is more convenient to use in integral form

$$\int_s \rho U dS = \text{const}. \tag{2}$$

We determined eddy viscosity by using the Buleev model [1], which is a variant of the Prandtl mixing-length model. This hypothesis allows us to write the equation for the transfer of thermal energy and mass in the form

$$\rho U c_p \frac{\partial T}{\partial z} = \frac{\partial}{\partial x}\left[(\lambda_f+\lambda_t)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\lambda_f+\lambda_t)\frac{\partial T}{\partial y}\right] + S_T; \tag{3}$$

$$\rho U\frac{\partial C_k}{\partial z} = \frac{\partial}{\partial x}\left[(D_k+D_t)\frac{\partial C_k}{\partial x}\right] + \frac{\partial}{\partial y}\left[(D_k+D_t)\frac{\partial C_k}{\partial y}\right] + R_k. \tag{4}$$

In the general case, the theoretical region includes the heat-conducting wall, while the system of transport equations is augmented by the heat conduction equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0. \tag{5}$$

Equations (1)-(5) can be represented in the form of the generalized diffusion equation

The coefficients A, Γ_x, and Γ_y in Eq. (6) and the source term Q are determined on the basis of the physical meaning of Eq. (6). The calculation of these coefficients is a fairly difficult procedure for the chemically reactive heat carrier N_2O_2–NO, in which the following reactions occur: 1) $N_2O_4 = NO_2$; 2) $2NO_2 = 2NO + O_2$; 3) $N_2O_3 = NO_2 + NO$. Since this issue is not central to the present problem, we will omit these calculations here. Calculations of the coefficients A, Γ_x, and Γ_y and the source terms S_T and R_k in Eqs. (1)-(4) are presented in [2]. We used methods such as those described in [3] to calculate the thermophysical and transport properties of the system N_2O_4–NO. We add only that the diffusion equation was solved for oxygen as a component in all of the chemical reactions that take place in the heat carrier. The distribution of the concentrations of the remaining components was determined with the assumption that the composition of the chemically reacting gas was stoichiometric at each moment of time. This assumption is quite valid for the given conditions. In the case of a gas flow of nonstoichiometric composition, it would be necessary to solve the diffusion equations for each component in the chemical reaction.

Boundary Conditions. The thermal boundary conditions are assigned on the outside surface of a rectangular tube (Fig. 1). Here, we assume the use of conditions of the first, second, and third types, as well as any combination of them.

On the inside surface of the channel walls, the conditions relating the temperatures to the heat fluxes are assigned

$$T_{E_2-0} = T_{E_2+0}; \quad \lambda_{wa} \left.\frac{\partial T}{\partial n}\right|_{E_2-0} = \lambda_b \left.\frac{\partial T}{\partial n}\right|_{E_2+0}.$$

For the equation of motion, the condition of adhesion to the washed part of the channel perimeter E_1 is imposed:

$$U_{E_1} = 0.$$

For all functions being studied, the condition of symmetry on the symmetry lines E_2 is satisfied (if such lines exist)

$$\left.\frac{\partial U}{\partial n}\right|_{E_2} = \left.\frac{\partial T}{\partial n}\right|_{E_2} = \left.\frac{\partial C_k}{\partial n}\right|_{E_2} = 0,$$

while for the diffusion equation we require satisfaction of the condition of impermeability on the inside wall

$$\left.\frac{\partial C_k}{\partial n}\right|_{E_1} = 0.$$

At the inlet of the channel (at z = 0), we prescribed uniform (plane) profiles of the functions V, T, and C_k.

By making appropriate use of the boundary conditions (fourth type), we can model not only convective but also radiative heat exchange between the tube and the environment. It should be noted that one of the advantages of the finite elements method is the possibility of creating a block program that permits rapid modification of the basic modules to solve the widest possible ranges of physical problems.

Turbulence Model. Eddy viscosity μ_t in Eqs. (2)-(4) is determined on the basis of the Buleev model. In accordance with the latter, the eddy viscosity coefficients have the following form:

$$A \frac{\partial \Phi}{\partial z} = \frac{\partial}{\partial x}\left(\Gamma_x \frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma_y \frac{\partial \Phi}{\partial y}\right) + Q. \tag{6}$$

$$\frac{\varepsilon_t^U}{\varepsilon} = 0{,}2 f_0(\eta) f_1(\eta) \gamma^*; \tag{7}$$

$$\frac{\varepsilon_t^T}{\varepsilon} = 0{,}2 f_0(\lambda\eta) f_1(\lambda\eta) \gamma^*; \tag{8}$$

$$f_0(\eta) = \exp(-\eta); \tag{9}$$

$$f_1(\eta) = [1 - \exp(-\eta)]/\eta; \tag{10}$$

$$\eta = 65/\gamma^*; \tag{11}$$

$$\lambda = \begin{cases} 0{,}8 + 0{,}2/\mathrm{Pr}^{0{,}67}, & \mathrm{Pr} \leqslant 1, \\ 1, & \mathrm{Pr} > 1, \end{cases} \tag{12}$$

$$\gamma^* = \frac{L^2}{\nu}\left|\frac{\partial U}{\partial n}\right|; \quad \left|\frac{\partial U}{\partial n}\right| = \sqrt{\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}. \tag{13}$$

In the general case, the turbulence scale for channels with a constant cross section is determined thusly:

$$\frac{1}{L} = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{l(\varphi)}\, d\varphi, \tag{14}$$

where $l(\varphi)$ is the distance from the point being examined to the channel walls in the direction determined by the angle φ. To calculate L, it is necessary to integrate the reverse distances from the point to the perimeter of the channel cross section. For a channel of rectangular cross section, the expression for the turbulence scale has the following form:

$$L = \frac{2x_1y_1x_2y_2}{x_2y_2\sqrt{x_1^2+y_1^2} + x_1y_2\sqrt{x_2^2+y_1^2} + x_2y_1\sqrt{x_1^2+y_2^2} + x_1y_1\sqrt{x_2^2+y_2^2}}, \tag{15}$$

where x_1 and x_2 are the distances of the point from one pair of opposite sides of the rectangle in the channel cross section; y_1 and y_2 are the distances of the point from the other pair of sides. The maximum value of L_c at the center of a channel with the sides 2a and 2b is accordingly

$$L_c = \frac{ab}{2\sqrt{a^2+b^2}}. \tag{16}$$

This value is convenient to use when comparing the dimensionless hydrodynamic and thermophysical characteristics of a flow.

Numerical Method of Solution. The use of the method of finite elements in conjunction with the Galerkin method leads to a system of nodal equations that can be written as follows in matrix form:

$$[C]\frac{d\{\Phi\}}{dz} + [K]\{\Phi\} + \{F\} = 0. \tag{17}$$

The contribution of each element to the matrices [K], [C], and {F} is expressed by the formulas:

$$[C]^{(e)} = \int_\Omega A[N]^T[N]\, d\Omega; \tag{18}$$

$$[K]^{(e)} = \int_\Omega [B]^T[D][B]\, d\Omega + \int_{S_2} \varkappa [N]^T[N]\, dS; \tag{19}$$

$$\{F\}^{(e)} = -\int_\Omega Q[N]^T d\Omega + \int_{S_1} q[N]^T dS - \int_{S_2} \varkappa\Phi_\infty [N]^T dS \tag{20}$$

[S_1 is the surface where we have assigned boundary conditions of the second type, i.e., $q = -k(\partial\Phi/\partial n)$, while S_2 is the surface on which we have assigned boundary conditions of the third type (the heat-transfer coefficient κ for the energy equation)]. Boundary conditions with a zero heat flux (adiabatic wall) are automatically considered in the finite elements methods. The global stiffness matrix [K] is obtained by summation over all of the elements in the usual manner. We similarly obtain the damping matrix [C] and the global load vector {F}. The matrix $[N]^T$ is the transpose of the matrix [N], while [B] and $[B]^T$ are the normal and transposed gradient matrices and [D] is the conductivity matrix.

The equations were integrated downflow (along the z axis) in accordance with an implicit two-layer scheme

$$K_{i+1}\Phi_{i+1} + C_{i+1}(\Phi_{i+1} - \Phi_i)/\Delta Z - F_{i+1} = 0. \tag{21}$$

As the calculations showed, combining the finite elements method with the finite differences method makes it possible to use two-dimensional elements and employ a scheduling procedure in integrating over z. Compared to the use of three-dimensional elements, this significantly reduces processor operating time and the amount of internal storage required. We use the direct Gauss method to solve the system of algebraic equations. The matrices were transformed to the band type.

TABLE 1. Values of Heat Flux $q_{wa}(x)$ and Shear Stress $\tau_{wa}(x)$ on the Walls of the Channel T_1 and T_2 with Different Values z/d_e

Value of the coordinate $x \cdot 10^2$, m	$q_{wa}(x) = -\lambda \frac{\partial T}{\partial y}$, W/m^2		$\tau_{wa}(x) = -\mu \frac{\partial U}{\partial y} \cdot 10^2$, N/m^2		z/d_e
	on the wall with T_1 = 500 K	with T_2=400 K	with T_1=500 K	with T_2=400 K	
0,0	724	12074	0,0	0,0	
0,1	729	12356	2,24	0,0190	
0,2	727	12316	2,39	0,0203	
1,0	693	11776	2,28	0,0194	
2,0	689	11712	2,26	0,0193	
3,0	688	11698	2,26	0,0193	0,04
4,0	689	11712	2,26	0,0193	
5,0	693	11776	2,28	0,0194	
5,8	727	12316	2,39	0,0203	
5,9	729	12356	2,24	0,0190	
6,0	724	12074	0,0	0,0	
0,0	487	1237	0,0	0,0	
0,1	497	1262	0,7	0,52	
0,2	485	1235	0,87	0,64	
1,0	519	1337	1,20	0,91	
2,0	533	1392	1,31	1,0	70,0
3,0	537	1410	1,34	1,0,	
4,0	533	1392	1,31	1,0	
5,0	519	1337	1,20	0,91	
5,8	485	1235	0,87	0,64	
5,9	497	1262	0,7	0,52	
6,0	487	1237	0,0	0,0	

The program has been widely used and no problems connected with convergence or the stability of the solution have arisen. The program was tested and carefully checked by comparison with experimental results, by electrical modeling, and by comparison of numerical results with analytical calculations. For example, in calculating heat transfer on the initial section of a circular pipe, the difference between the theoretical and experimental values of temperature did not exceed 2%.

Results and Discussion. Calculations were performed for a turbulent gas flow in a channel with 0.06 × 0.06 m sides. The velocity, temperature, and relative concentration of oxygen at the channel inlet were as follows: U = 1 m/sec; T_0 = 485 K; $c_{k0} = 10^{-3}$. The thermal boundary conditions (on the outside surface of the tube) were either fixed temperatures T_1 and T_2 equal to 400 and 500 K for the top and bottom walls, respectively, and adiabatic conditions on the lateral surfaces or fixed temperatures T_1 and T_2 for the bottom and top walls, respectively, and a linear change in the temperatures of the side walls. Conditions of impermeability on the inside walls of the channel were assigned for the diffusion equation.

Table 1 shows calculated local values of heat flux $q_{wa}(x)$ and shear stress $\tau_{wa}(x)$.

It was once thought [4] that heat transfer in complex channels for heat carriers with Prandtl numbers greater than unity could be described by the formulas for circular tubes with the use of the equivalent hydraulic diameter in the similarity criteria. The experiments in [4] showed this notion to be false. For example, in an examination of heat transfer in tube bundles, it turned out that the perimeter-averaged values of Nu with dense packing were roughly half the values calculated from the Ditus–Bolter and Mikheev formulas for circular tubes. With an increase in the mesh of the subdivision, the relative numbers Nu exceeded the results for circular tubes by 30-35% [5].

The feasibility of using Mikheev's criterional formula to calculate heat transfer in a rectangular channel was checked by comparing the results of two series of calculations. The first was obtained using the criterional formula, while the second was obtained from the differential model. Here, the mean value of the number $\overline{Nu}$ was determined from the relation:

$$\overline{Nu} = \frac{\bar{q}_{wa}(x)\, d_e}{(\bar{T}(x) - \bar{T}_b)\,\lambda_b}. \qquad (22)$$

It turned out that the criterional method yielded higher (by 20-30%) values.

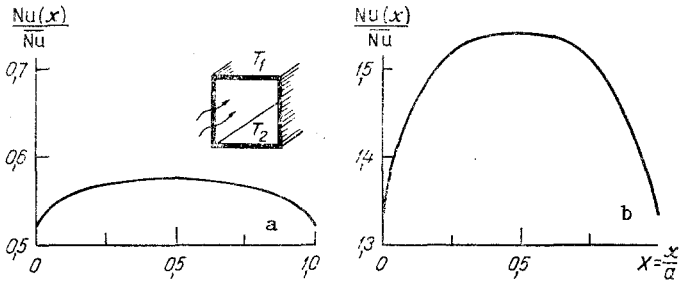

Fig. 2. Distribution of local numbers Nu(x) on the sides of a rectangular tube: a) along the wall at T_1 = 400 K; b) at T_2 = 500 K, z/d = 70.

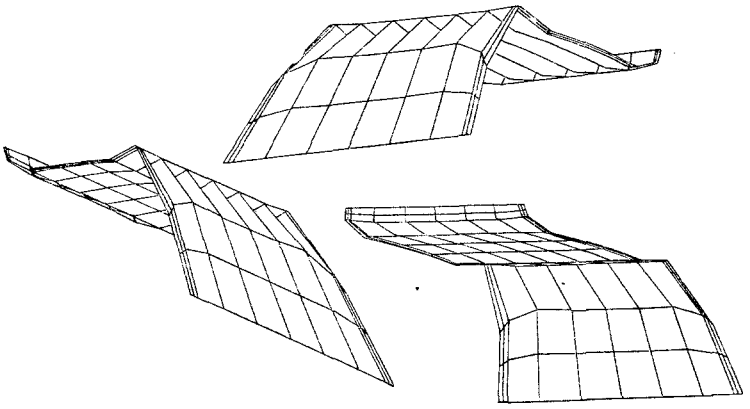

Fig. 3. Temperature surfaces (central projection) in a rectangular channel with different foreshortenings.

Besides calculating the mean values $\overline{Nu}$, we obtained the distributions of the local values Nu(x). These are shown in the form of a dimensionless function $f_{Nu} = Nu(x)/Nu$. The nonuniformity in the distribution f_{Nu} of the variants we studied reached 10-12%.

The overall pattern of distribution of the investigated functions in the channel is shown in Fig. 1 in the form of isolines of U, T, and C_{k0}.

The initial value of the step Δz was 10^{-5} m, while use of the procedure of automatic step selection made it possible to increase Δz to values of 0.01-0.3 m. Since most of the time spent on solving the initial system is spent on integrating the equation of motion, the velocity profile was calculated until the standard deviation of the velocity values on two "time" layers did not differ by more than 1%.

Conclusions. The modular structure of the program makes it possible, without significant modifications, to study channels of different configurations (tube bundles, annular channels, rectangular channels, etc.) and phenomena of different types (free and forced convection) and to consider factors such as the variability and anisotropy of thermophysical properties.

By resorting to direct numerical integration of the system of transport equations, it is possible to obtain almost complete information on the distribution of hydrodynamic and thermal characteristics for a flow. The use of graphical service programs makes it possible to represent the information in the form of three-dimensional figures (second-order curves), for example. In connection with this, Fig. 3 shows temperature surfaces with different foreshortenings. The coupled formulation of the problem makes it possible to avoid the difficulties connected with boundary conditions on the heat-transfer surface.

The use of criterional formulas to model heat transfer in rectangular channels is difficult in the case of combination boundary conditions, while the use of Mikheev formulas leads to sizable errors (as shown by the above calculations).

The use of a hybrid method combining finite element calculation of the parameters in the channel cross section (the use of planar elements) and finite-difference approximation lengthwise is evidently the most effective and expedient approach with regard to saving computer storage and cutting processor operating time. The latter is 30-40 sec for one step on an ÉS-1061 computer.

NOTATION

U, velocity; T, temperature; C_k, concentration of the k-th component; x, y, z, cartesian coordinate system; ρ, density; λ_f, frozen thermal conductivity; μ_t, λ_t, and D_t, turbulent values of the viscosity coefficient, thermal conductivity, and diffusion, respectively; $\overline{Nu}$, mean Nusselt number for both sides T_1 and T_2. Indices: (e) – for an element; wa – wall.

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APPROXIMATE SOLUTION OF A PROBLEM OF CONVECTIVE HEAT TRANSFER BETWEEN A PLATE AND LIQUID METALS

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This article examines a theoretical method of calculating the heat-transfer coefficient for different values of the Reynolds number of a liquid-metal flow onto a plate.

The differential (local) method has become the method most commonly used in the general theory of qualitative and quantitative description of heat transfer under conditions of wall turbulence. In this method, turbulent heat transfer is completely determined by the physical parameters (density, viscosity, distributions of mean velocities and temperature) of a uniform fluid flow (liquid metals, gas, liquids in drop form) [1]. If we connect a translating coordinate system with a local fluid particle, then in accordance with Galileo's principle all of the dynamic processes of turbulent transport will occur identically in regard to this inertial system of reference [1].

Let only two physical quantities – momentum and heat – be transported through streamlines representing the averaged motion of the fluid medium. Then the transfer of momentum creates turbulent friction between the layers of the fluid, while heat transfer results in turbulent heat conduction. Since there are no other factors contributing to turbulent heat transfer in the given case, the turbulent mixing mechanism will be the same for both turbulent friction and turbulent heat conduction [1]. Meanwhile, the same volumes of fluid simultaneously transfer momentum and heat. If no heat is exchanged with the environment, then it follows from the Prandtl theory [1] that if momentum is conserved, then the amount of heat transferred by the fluid volumes is also conserved. This leads to a situation whereby the turbulent Prandtl number, characterizing the connection between turbulent transfer of

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